TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

PreCalculus

Honors
Mathematics Department
Trumbull High School

2016

(Last revision date: 2007)

Curriculum Writing Team

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PreCalculus Honors

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CORE VALUES AND BELIEFS

The Trumbull School Community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION

Honors PreCalculus is an extensive course that uses and strengthens students' knowledge of algebra and geometry. Investigations of relations and functions help to build meaningful models of real-world situations. Students will be presented mathematical tasks that require the application of mathematics in new and unfamiliar situations and to real-world scenarios.

Honors PreCalculus is designed to develop the eight standards of mathematical practices in students. It will extend and refine advanced algebraic and trigonometric concepts and will introduce concepts in vectors, polars, sequences and series, limits, and basic differentiation.This course emphasizes problem solving through application and technology.

Graphing calculators will be used throughout the course to assist students with computation, problem solving, and visual connections. Instruction will be given on the correct use of the graphing calculator as a technological aid, a computational tool, and an accuracy check of solutions and graphs.

This rigorous course is designed to strengthen students' conceptual understanding of problems and mathematical reasoning in solving problems as well as prepare students who excel in mathematics for AP Calculus AB and BC and/or college coursework in Calculus. This course is aligned with the 2010 Connecticut Core Standards for Mathematics.

PHILOSOPHY

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.
The successful mathematician will:

- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process

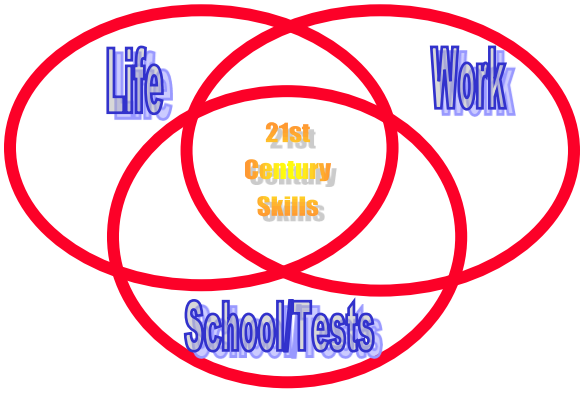
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourge students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media And Technology Skills

1. Use real-world digital and other research tools to access, evaluate and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE DESCRIPTION

Honors PreCalculus is intended to review and strengthen skills necessary for an AP calculus course. Topics include: functions, math modeling, inequalities, exponents, logarithms, trigonometry, sequences, series, limits, and an introduction to Calculus. A graphing calculator is required for this course.

GOALS

The Standards for Mathematical Practice describe varieties of expertise that all teachers of mathematics will develop in their students.

These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

COURSE SYLLABUS

Course Name

Honors PreCalculus

Level

Honors

Prerequisites

Completion of Honors Algebra II with a B- or better and teacher recommendation.

Materials Required

TI-84 graphing calculator

General Description of the Course

Honors PreCalculus is intended to review and strengthen skills necessary for an AP calculus course. Topics include: functions, math modeling, inequalities, exponents, logarithms, trigonometry, sequences, series, limits, and an introduction to Calculus. A graphing calculator is required for this course.

Assured Assessments

Students will be evaluated by their performance on tests, quizzes, homework, problem sets, journals, APBAs, and departmental midterm and final examinations.

Core Text

Precalculus with Limits: A Graphing Approach, Brooks, 2011

Unit 1: Functions

Performance Standards

The Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

A-APR Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-BF Building Functions

Build new functions from existing functions.

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Essential Questions

- How are the graphs of functions affected by coefficients and additions?
- How can rate of change be best represented mathematically?

- How does the domain impact the solutions?
- How can an expression be simplified using the properties of exponents?
- How is symmetry determined?
- How can symmetry be used to graph a relation?
- How is the domain of function determined?
- How is a graph related to its parent graph using transformations?
- How is a piecewise function graphed?
- What is the relationship of a piecewise graph and specific domains?
- What is function notation and how are functions evaluated?
- How are difference quotients evaluated?
- How is an inverse function graphed?
- How is an inverse function found algebraically?
- What is the process to prove two functions are inverses?
- On which interval are functions increasing or decreasing and determine relative maximum and relative minimum values of functions?
- How do we identify odd and even functions?
- How is the average rate of change of a function found?

Content (Scope and Sequence)

Properties of exponents
Symmetry
Properties of functions
Graphs of functions
Parent functions
Piecewise functions
Transformations of functions
Composition of functions
Inverse functions

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Summer Packet
Graphing Calculator Tour Activity
Journal Questions

Time Allocation

Approximately 4 weeks

Unit 2: Polynomial and Rational Functions

Performance Standards

The Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-CN The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeroes of the function it defines.
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A-APR Arithmetic with Polynomials and Rational Expressions

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI Reasoning with Equations and Inequalities

Solve equations and inequalities in one variable.

4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Essential Questions

- How do quadratic and cubic functions model non-linear behavior?
- How does one determine which method is best for dividing polynomials?
- How does one write a rational expression as the sum of two or more simpler rational expressions?
- How does one analyze graphs of quadratic functions?
- How does one write quadratic functions in standard form and use the results to sketch graphs of functions?

- How does one find the minimum and maximum values of quadratic functions in real-life applications?
- How can one use transformations to sketch graphs of polynomial functions?
- What is the Leading Coefficient Test, and how does one use it to determine end behavior of graphs of polynomial functions?
- How are zeros of polynomial functions used as sketching aids?
- What is the Intermediate Value Theorem and how is it used to help locate zeros of polynomial functions?
- How does one use long and synthetic division to divide polynomials?
- What is the difference between the Remainder Theorem and the Factor Theorem?
- How does one use complex conjugates to write the quotient of two complex numbers in standard form?
- How does one find complex solutions of quadratic equations?
- How can the Fundamental Theorem of Algebra be used to determine the number of zeros of polynomial functions?
- How does one find rational zeros of polynomial functions?
- How can one find conjugate pairs of complex zeros?
- How can one find zeros of polynomials by factoring?
- How does one find the domains of rational functions?
- How does one find the vertical, horizontal, and slant asymptotes and points of discontinuity of rational functions?
- What key elements does one need to sketch the graph of a rational function?
- How can rational functions be used to model and solve real life problems?
- How does one solve polynomial and rational inequalities?
- How does one find the partial fraction decompositions of rational functions?

Content (Scope and Sequence)

Quadratic functions
Polynomial functions
Division of polynomials
Complex numbers
Factoring polynomials
Zeros of polynomials
Rational functions
Polynomial and rational inequalities
Partial fractions

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

TI-84 Activity: Breaking Up Is Not Hard To Do

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 5 weeks

Unit 3: Exponential and Logarithmic Functions

Performance Standards

The Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.01212^t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

F-IF Interpreting Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    e. Graph exponential and logarithmic functions, showing intercepts and end-behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    b. Use the properties of exponents to interpret expressions for exponential functions.

F-BF Building Functions

Build new functions from existing functions.

4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
4. For exponential models, express as a logarithm the solution to ab $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Essential Questions

- What is the relationship between exponential and logarithmic functions?
- How are exponential and logarithmic functions interpreted and graphed?
- What are the properties of logarithms?
- How are exponential and logarithmic equations solved?
- How are exponential equations used to model real-world situations?
- How does one recognize, evaluate, and graph natural logarithmic functions?
- How are properties of logarithms used to evaluate, re-write, expand or condense expressions, including the change of base formula?

Content (Scope and Sequence)

Exponential functions & graphs
Exponential functions as mathematical models (compound interest)
Logarithmic functions & graphs
Properties of logarithms
Exponential equations
Logarithmic equations

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 3 weeks

Unit 4: Trigonometry

Performance Standards

The Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-SRT Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Essential Questions

- How are the properties of right triangles and trigonometry used in the real world?
- How do periodic functions relate to applications in the real world?
- What are periodic functions?
- How are arc length and angle measures used on a unit circle?
- What is a reference angle and how is it found?
- What are the six trigonometric ratios?
- How are the six trigonometric ratios related to each other?
- What are circular functions?
- What are the six trigonometric functions of 30° and 45° angles?
- How do the cycle, period, and amplitude impact the graph of a circular function?
- What is a sinusoidal graph?
- How are inverse trigonometric ratios used to find angle measures of a right triangle?
- How is trigonometry used to find parts of a right triangle?
- What are angles of elevation and depression?
- How is trigonometry used to find the area of a triangle?
- Given *SSA*, what determines if there can be more than one solution in the triangle?

Content (Scope and Sequence)

Radian and degree measure
Arc length
Area of a sector
Unit circle
Right-triangle trigonometry
Trigonometric identities
Sinusoidal graphs
Period and amplitude of periodic functions
Translating trigonometric graphs
Inverse trigonometric functions

Applications of trigonometric function
Law of Sines
Law of Cosines
Area of oblique triangles
Simplifying and rewriting trigonometric expressions
Verifying trigonometric identities
Solving trigonometric equations
Sum and Difference Formulas

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

APBA Trigonometry Weather Project: The Impact of Weather on Travel Locations
Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 9 weeks

Unit 5: Vectors

Performance Standards

The Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-VM Vector and Matrix Quantities

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

Perform operations on vectors.

4. (+) Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
    c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

Essential Questions

- What is a vector?
- How does one represent and perform operations with vector quantities?
- How are vectors represented as directed line segments?
- How is the component form of a vector written?
- How are basic vector operations performed and how are they represented graphically?

Content (Scope and Sequence)

Representing vectors as directed line segments
Writing the component forms of vectors
Performing basic vector operations
Representing vectors graphically

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 1 week

Unit 6: Polar Coordinates, Parametric Functions

Performance Standards

At this point students are expected to able to demonstrate an understanding of all 2010 Connecticut Core Standards in Mathematics, including in number and quantity, algebra, functions, modeling, and geometry.

The TPS-created unit objective is:

- The student will write equations to describe the position of a point in a plane using distance and angle rather than x- and y-coordinates.

Essential Questions

- How are points plotted on the polar coordinate system?
- How are rectangular coordinates converted to polar form and vice versa?
- How are rectangular equations converted to polar form and vice versa?
- How are circles and lines graphed in the polar coordinate system?
- How can sets of parametric equations be evaluated for given values of the parameter?
- How are curves sketched that are represented by sets of parametric equations?
- How can sets of parametric equations be rewritten as a single rectangular equation?
- How does one find a set of parametric equations for a graph?

Content (Scope and Sequence)

Plot points, circles and lines on the polar coordinate system
Converting rectangular coordinates converted to polar form and vice versa
Converting rectangular equations converted to polar form and vice versa
Sketching a plane curve
Eliminating the parameter and angle parameter
Finding parametric equations for graphs

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

TI-84 Polar Graphing Activity
Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 2 weeks

Unit 7: Sequences and Series

Performance Standards

At this point students are expected to able to demonstrate an understanding of all 2010 Connecticut Core Standards in Mathematics, including in number and quantity, algebra, functions, modeling, and geometry.

The TPS-created unit objective is:

- The student will represent a sequence of numbers or the sum of a sequence of numbers.

Essential Questions

- How does one use sequence notation, factorial notation and summation notation?
- How does one find the sums of series?
- How does one recognize, write and find the nth terms of arithmetic and geometric sequences?
- How does one find nth partial sums of arithmetic and geometric sequences?
- How does one find the sum of an infinite geometric series?

Content (Scope and Sequence)

Writing the terms of a sequence, including those involving factorials
Finding the nth term of a sequence
Summation notation
Finding the sum or partial sum of a series
Finding the nth term of an arithmetic or geometric sequence
Using a recursion formula
Finding the sum of an infinite geometric series

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 2 weeks

Unit 8: Limits

Performance Standards

At this point students are expected to able to demonstrate an understanding of all 2010 Connecticut Core Standards in Mathematics, including in number and quantity, algebra, functions, modeling, and geometry.

The TPS-created unit objective is:

- The student will calculate and interpret limits of functions, including limits that cannot be solved by direct substitution, limits at infinity, and limits of sequences.

Essential Questions

- What is a limit?
- How does continuity depend on limits?
- How does one estimate a limit using a numerical or graphical approach?
- What are the different ways that a limit can fail to exist?
- How does one evaluate a limit using properties of limits?
- What strategies can one use for finding limits?
- How does one evaluate a limit using dividing out and rationalizing techniques?
- How does one determine continuity at a point and continuity on an open interval?
- How does one determine one-sided limits and continuity on a closed interval?
- How does one use the properties of continuity?
- How does one determine infinite limits from the left and from the right?
- How does one determine finite and infinite limits at infinity?
- How does one find limits of sequences?
- How does one find limits of summations?
- How does one use rectangle to approximate areas of plane regions?
- How does one find the exact area of plane regions?

Content (Scope and Sequence)

Finding limits graphically, numerically, analytically, algebraically
Continuity and one-sided limits
Infinite limits and limits at infinity
Limits of sequences
Limits of summations
Area of plane regions

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
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Assured Experiences (Projects)

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 3 weeks

Unit 9: The Derivative

Performance Standards

At this point students are expected to able to demonstrate an understanding of all 2010 Connecticut Core Standards in Mathematics, including in number and quantity, algebra, functions, modeling, and geometry.

The TPS-created unit objective is:

- The student will find the slope of a graph at any single point.

Essential Questions

- How does the concept of a limit lead to a derivative?
- How does one determine the derivative of a function?
- What is the relationship between differentiability and continuity?
- What is a derivative?
- How does one find the derivative of a function using rules?
- How can one use the derivative to find the velocity and acceleration of a projectile?

Content (Scope and Sequence)

The tangent line problem

Average and instantaneous rates of change

Definition of the derivative

Continuity

Finding equations of tangent lines to a curve at a given point

Finding derivatives explicitly using sum, constant, power rules, and trigonometric functions

Instructional/Teaching Strategies

Whole-class instruction, guided practice, modeling, collaborative work, homework groups, brainstorming, individual, partner or group journal writing, problem sets, interactive activities, graphing calculator demonstration and exploration, student explanation of solutions, and study guides

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
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5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Tests, Quizzes, Problem Sets, Homework, Journal Questions

Time Allocation

Approximately 2 weeks

TEACHER GUIDE

Unit One: Functions

Sections:

1.2 Graphs of Equations
1.4 Functions
1.5 Analyzing Graphs of Functions
1.6 A Library of Parent Functions
1.9 Inverse Functions

Unit Two: Polynomial and Rational Functions

2.1 Quadratic Functions and Models
2.2 Polynomial Functions of Higher Degree
2.3 Polynomial and Synthetic Division
2.4 Complex Numbers
2.5 Zeros of Polynomial Functions
2.6 Rational Functions
2.7 Nonlinear Inequalities
7.4 Partial Fractions

Unit Three: Exponential and Logarithmic Functions

3.1 Exponential Functions and Their Graphs
3.2 Logarithmic Functions and Their Graphs
3.3 Properties of Logarithms
3.4 Exponential and Logarithmic Equations

Unit Four: Trigonometry

4.1 Radian and Degree Measure
4.2 Trigonometric Functions: The Unit Circle
4.3 Right Triangle Trigonometry
4.4 Trigonometric Functions of Any Angle
6.1 Law of Sines
6.2 Law of Cosines
4.5 Graphs of Sine and Cosine Functions
4.6 Graphs of Other Trigonometric Functions
4.7 Inverse Trig Functions
4.8 Applications and Models

***Midterm exam**

5.1 Using Fundamental Identities
5.2 Verifying Trigonometric Identities
5.3 Solving Trigonometric Equations
5.4 Sum and Difference Formulas
5.5 Double and Half Angle Formulas

Unit Five: Vectors

6.3 Vectors in the Plane

Unit Six: Polar Coordinates, Parametric Functions

10.6 Parametric Equations
10.7 Polar Coordinates

Unit Seven: Sequences and Series

9.1 Series and Sequences
9.2 Arithmetic Sequences and Partial Sums
9.3 Geometric Sequences and Series

Unit Eight: Limits

12.1 Introduction to Limits
12.2 Techniques for Evaluating Limits
12.4 Limits at Infinity and Limits of Sequences
12.5 The Area Problem

Unit Nine: The Derivative

12.3 The Tangent Line Problem
Supplement: Average and Instantaneous Rate of Change
Continuity
Finding equations of tangent lines to a curve at a given point
Finding derivatives explicitly using sum, constant, power rules
Trigonometric functions

COURSE CREDIT

One credit in Mathematics

One class period daily for a full year

PREREQUISITES

Completion of Honors Algebra II with a B- or better and teacher recommendation.

TEXT

Precalculus with Limits: A Graphing Approach, Brooks, 2011

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department- and teacher-prepared materials

TI-84 Plus graphing calculators

CURRENT REFERENCES

2010 Connecticut Core Standards for Mathematics

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric
- Trumbull High School School-Wide Problem-Solving Rubric
- Trumbull High School School-Wide Independent Learning and Thinking Rubric

Rubric 2: Write Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_____	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_____	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_____	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors. Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Rubric 3: Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task	Student demonstrates sufficient understanding of the problem and most of the complexities of the task	Student demonstrates some understanding of the problem but requires assistance to complete the task	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal	Student gathers sufficient information from multiple sources including digital, print, and interpersonal	Student gathers some information from few sources including digital, print, and interpersonal	Student gathers limited or no information
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

Rubric 5: Independent Learners And Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	Presentation shows compelling evidence of an independent learner and thinker. Solution shows deep understanding of the problem and its components. Solution shows extensive and appropriate application of 21st Century Skills.	Presentation shows clear evidence of an independent learner and thinker. Solution shows adequate understanding of the problem and its components. Solution shows adequate application of 21st Century Skills.	Presentation shows some evidence of an independent learner and thinker. Solution shows some understanding of the problem and its components. Solution shows some application of 21st Century Skills.	Presentation shows limited or no evidence of an independent learner and thinker. Solution shows limited or no understanding of the problem. Solution shows limited or no application of 21st Century Skills.